Numerical study of a biped equipped of a lower-limb power-assist exoskeleton

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LMA Seminar

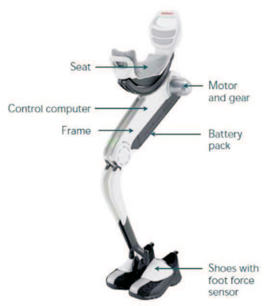
October 16, 2014

Objectives

- To consider a planar Biped with a wearable assist device.
- Ballistic walking gait (no applied finite torques during the motion).
- Why to study the ballistic motion? The ballistic walking may be close to the human walking gait during the swing phase. To improve our knowledge about a new bipedal robot
- Infinity of solutions to find impulsive torques.
- Minimization of an energy cost functional based on these impulsive torques.

Un exemple: Le prototype Honda

Utilisation dans un milieu industriel



Generalized coordinates and torques

- 15 generalized coordinates $\mathbf{x} = [q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_{91}, q_{92}, q_{101}, q_{102}, q_{11}, x, y]^\top$.
- q_8, q_{91}, q_{92}, q_{101}, q_{102}, and q_{11} are devoted to the wearable assist device.
- Torque vector, Γ number of components between six (no assistance) and 12 (fully assisted).

The geometrical structure

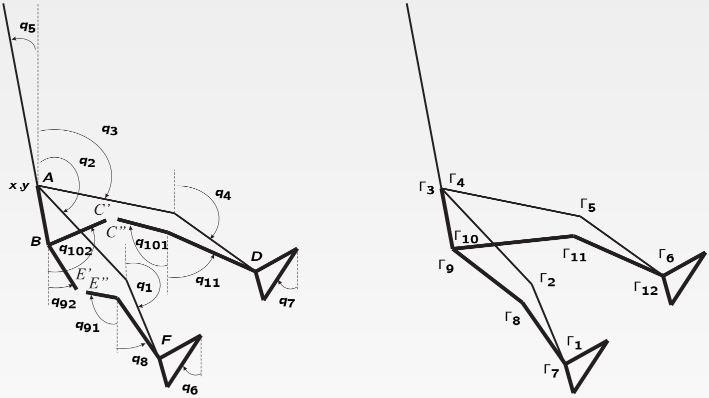


Figure : The geometrical structure with its DoF and link frames of the biped and its walking assist device.

Physical parameters.

	Mass (kg)	Length (m)	Inertia ($kg \cdot m^2$)	center of mass (m)
Foot and shoe	$m_f = 0.678$	$L_p = 0.207$ $l_p = 0.072$ $H_p = 0.064$	$I^f = 0.012$	$s_{px} = 0.0135$ $s_{py} = 0.0321$
Shin	$m_s = 4.6$	$l_s = 0.497$	$I^s = 0.0521$	$s_s = 0.324$
Thigh	$m_t = 8.6$	$l_t = 0.41$	$I^t = 0.7414$	$s_t = 0.18$
Trunk	$m_T = 16.5$	$l_T = 0.625$	$I^T = 11.3$	$s_T = 0.386$
Seat	$m_3 = 2.0$	$l_3 = 0.1$	$I^T = 0.3$	$s_3 = 0.05$
Upper frame	$m_1 = 3.0$	$l_1 = 0.392$	$I^1 = 0.04$	$s_1 = 0.1127$
Lower link	$m_2 = 2.0$	$l_2 = 0.3645$	$I^2 = 0.02$	$s_2 = 0.169$

Table : Physical parameters of the seven-link biped and of its walking assist device.

Matrix equations

Equations of the motion of the biped in single support

$$\mathbf{A}(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{h}(\mathbf{x}, \dot{\mathbf{x}}) = \begin{bmatrix} \mathbf{D} & \mathbf{J}_1^\top & \mathbf{J}_2^\top \end{bmatrix} \begin{bmatrix} \mathbf{\Gamma} \\ \mathbf{f}_c \end{bmatrix} + \mathbf{J}_{\mathbf{r_1}}^\top \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{m}_{1_z} \end{bmatrix} + \mathbf{J}_{\mathbf{r_2}}^\top \begin{bmatrix} \mathbf{r}_2 \\ \mathbf{m}_{2_z} \end{bmatrix}, \quad (1)$$

with the constraint equations,

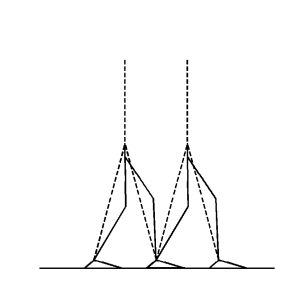
$$\begin{gathered} \mathbf{J}_{\mathbf{r}_i}\ddot{\mathbf{x}} + \dot{\mathbf{J}}_{\mathbf{r}_i}\dot{\mathbf{x}} = \mathbf{0} \text{ for } i = 1 \text{ to } 2, \\ \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \end{bmatrix} \ddot{\mathbf{x}} + \begin{bmatrix} \dot{\mathbf{J}}_1 \\ \dot{\mathbf{J}}_2 \end{bmatrix} \dot{\mathbf{x}} = \mathbf{0}. \end{gathered} \quad (2)$$

$\begin{bmatrix} \mathbf{r}_i & \mathbf{m}_{i_z} \end{bmatrix}^\top$, with $i = 1$ to 2, : resultant wrenches of the contact efforts
$\boldsymbol{\lambda}_i = \mathbf{f}_{c_i} = [f_{x_i}, f_{y_i}, m_{z_i}]^\top$: the wrench, for each loop closure.

Definition of the ballistic motion

Statement of the problem

- Let $\mathbf{x}(0)$ be an initial configuration $t = 0$.
- Let $\mathbf{x}(T)$ be an final configuration $t = T$.



Definition of the ballistic motion

ballistic motion be in single support: $\Gamma = 0$.

$$\mathbf{A}(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{h}(\mathbf{x}, \dot{\mathbf{x}}) = \begin{bmatrix}\mathbf{D} & \mathbf{J}_1^\top & \mathbf{J}_2^\top\end{bmatrix} \begin{bmatrix}\mathbf{0}_{12\times 0} \\ \mathbf{f}_c\end{bmatrix} + \mathbf{J}_{\mathbf{r_1}}^\top \begin{bmatrix}\mathbf{r}_1 \\ \mathbf{m}_{1_z}\end{bmatrix} + \mathbf{J}_{\mathbf{r_2}}^\top \begin{bmatrix}\mathbf{r}_2 \\ \mathbf{m}_{2_z}\end{bmatrix}$$

$$\mathbf{J}_{\mathbf{r}_i}\ddot{\mathbf{x}} + \dot{\mathbf{J}}_{\mathbf{r}_i}\dot{\mathbf{x}} = \mathbf{0} \text{ for } i = 1 \text{ to } 2,$$

$$\begin{bmatrix}\mathbf{J}_1 \\ \mathbf{J}_2\end{bmatrix}\ddot{\mathbf{x}} + \begin{bmatrix}\dot{\mathbf{J}}_1 \\ \dot{\mathbf{J}}_2\end{bmatrix}\dot{\mathbf{x}} = \mathbf{0}.$$

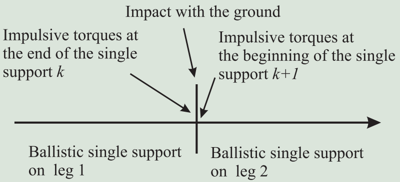
- problem: Which velocity vector $\dot{\mathbf{x}}(0)$ such that $\mathbf{x}(t)$ starting from $\mathbf{x}(0)$ reaches $\mathbf{x}(T)$

$\Rightarrow$ A boundary value problem solved using a Newton method

Impulsive control

decomposition of the impulsive impact.

- After impact the velocity of the biped has to be equal to the founded initial velocity. $\Rightarrow$ impulsive impact.



- Let $\dot{\mathbf{x}}^b$ be the final velocity vector of the current ballistic swing,
- Let $\dot{\mathbf{x}}^a$ be the initial velocity vector of the next ballistic swing.

The structure of the instantaneous double support

First sub-phase: Impulsive phase.

$$\mathbf{A}[\mathbf{x}(T)](\dot{\mathbf{x}}^- - \dot{\mathbf{x}}^b) = \begin{bmatrix}\mathbf{D} & \mathbf{J}_1^\top & \mathbf{J}_2^\top\end{bmatrix} \begin{bmatrix}\mathbf{I}^- \\ \mathbf{I}_{\mathbf{f_c}}^-\end{bmatrix} + \mathbf{J}_{\mathbf{r_1}}^\top \mathbf{I}_{\mathbf{r_1}}^-. \quad (3)$$

$$\mathbf{J}_{\mathbf{r_1}} \dot{\mathbf{x}}^- = \mathbf{0}_{3\times 1}$$

$$\begin{bmatrix}\mathbf{J}_1 \\ \mathbf{J}_2\end{bmatrix} \dot{\mathbf{x}}^- = \mathbf{0}_{6\times 1}. \quad (4)$$

24 scalar equations.

The structure of the instantaneous double support

Second sub-phase: Passive impact.

The *second* sub-phase: passive impact. The stance leg lifts off the ground (Hypothesis).

$$\mathbf{A}\left(\dot{\mathbf{x}}^{+} - \dot{\mathbf{x}}^{-}\right) = \begin{bmatrix}\mathbf{D} & \mathbf{J}_1^\top & \mathbf{J}_2^\top\end{bmatrix} \begin{bmatrix}\mathbf{0}_{n_a \times 1} \\ \mathbf{I}_{\mathbf{f_c}}\end{bmatrix} + \mathbf{J}_{r_2}^\top \mathbf{I}_{\mathbf{r_2}} \tag{5}$$

The associate equation:

$$\begin{bmatrix}\mathbf{J}_1 \\ \mathbf{J}_2\end{bmatrix} \dot{\mathbf{x}}^{+} = \mathbf{0}_{6\times 1}. \tag{6}$$

$$\mathbf{J}_{\mathbf{r_2}} \dot{\mathbf{x}}^{+} = \mathbf{0}_{3\times 1} \tag{7}$$

24 scalar equations.

The structure of the instantaneous double support

Third sub-phase: Impulsive impact.

$$\mathbf{A}\left(\dot{\mathbf{x}}^{a}-\dot{\mathbf{x}}^{+}\right)=\left[\mathbf{D} \quad \mathbf{J}_{1}^{\top} \quad \mathbf{J}_{2}^{\top}\right]\begin{bmatrix}\mathbf{I}^{+}\\ \mathbf{I}_{\mathbf{f_c}}^{+}\end{bmatrix}+\mathbf{J}_{\mathbf{r_2}}^{\top}\mathbf{I}_{\mathbf{r_2}}^{+} \tag{8}$$

15 scalar equations.

$\Rightarrow$ 63 scalar equations, $69+n_a$ unknown variables:
$\dot{\mathbf{x}}^{-}$, $\mathbf{I}^{-}(n_a \times 1)$, $\mathbf{I}_{\mathbf{f_c}}^{-}$, $\mathbf{I}_{\mathbf{r_1}}^{-}$ (for the *first* sub-phase),
$\dot{\mathbf{x}}^{+}$, $\mathbf{I}_{\mathbf{f_c}}$, $\mathbf{I}_{\mathbf{r_2}}$ (for the *second* sub-phase),
$\mathbf{I}^{+}(n_a \times 1)$, $\mathbf{I}_{\mathbf{f_c}}^{+}$ and $\mathbf{I}_{\mathbf{r_2}}^{+}$ (for the *third* sub-phase).

$\Rightarrow$ Among this set of $69+n_a$ unknown variables, $6+n_a$ can be defined as optimization variables to minimize a cost functional.

Three Criteria

First Criterion:

for the biped

$$\mathcal{W}_1^b = \sum_{i=1}^{6} \left[\int_{T-}^{T} \left| \Gamma_i^-(t)\dot{\theta}_i(t) \right| \mathrm{d}t + \int_{T}^{T+} \left| \Gamma_i^+(t)\dot{\theta}_i(t) \right| \mathrm{d}t \right] \tag{9}$$

for the assist device

$$\mathcal{W}_1^{ad} = \sum_{i=7}^{12} \left[\int_{T-}^{T} \left| \Gamma_i^-(t)\dot{\theta}_i(t) \right| \mathrm{d}t + \int_{T}^{T+} \left| \Gamma_i^+(t)\dot{\theta}_i(t) \right| \mathrm{d}t \right] \tag{10}$$

with

$$\begin{array}{llll} \theta_1 = q_1 - q_6, & \theta_2 = q_2 - q_1, & \theta_3 = q_5 - q_2, & \theta_4 = q_3 - q_5, \\ \theta_5 = q_4 - q_3, & \theta_6 = q_7 - q_4, & \theta_7 = q_8 - q_6, & \theta_8 = q_{91} - q_8, \\ \theta_9 = q_5 - q_{92}, & \theta_{10} = q_{102} - q_5, & \theta_{11} = q_{11} - q_{101}, & \theta_{12} = q_7 - q_{11}. \end{array}$$

Calculation of the functional Cost

The energy due to the impulsive torques becomes [Formal 82]

$W = \sum_{i=1}^{n_a} \left(W_i^- + W_i^+\right)$ with:

$$W_i^- = \left| I_i^- \frac{\dot\theta_i(T^-) + \dot\theta_i(T)}{2} \right| \quad \text{if} \quad \dot\theta_i(T^-)\dot\theta_i(T) \geq 0,$$

$$W_i^- = \left| I_i^- \frac{\dot\theta_i^2(T^-) + \dot\theta_i^2(T)}{2\left[\dot\theta_i(T^-) - \dot\theta_i(T)\right]} \right| \quad \text{if} \quad \dot\theta_i(T^-)\dot\theta_i(T) < 0,$$

$$W_i^+ = \left| I_i^+ \frac{\dot\theta_i(T) + \dot\theta_i(T^+)}{2} \right| \quad \text{if} \quad \dot\theta_i(T)\dot\theta_i(T^+) \geq 0,$$

$$W_i^+ = \left| I_i^+ \frac{\dot\theta_i^2(T) + \dot\theta_i^2(T^+)}{2\left[\dot\theta_i(T) - \dot\theta_i(T^+)\right]} \right| \quad \text{if} \quad \dot\theta_i(T)\dot\theta_i(T^+) < 0$$

Three Criteria

Second Criterion:

- for the biped

$$\mathcal{W}_2^b = \sum_{i=1}^{6} \left[\int_{T-}^{T} \left|\Gamma_i^-(t)\right| \mathrm{d}t + \int_{T}^{T+} \left|\Gamma_i^+(t)\right| \mathrm{d}t \right]. \tag{11}$$

- for the assit device

$$\mathcal{W}_2^{ad} = \sum_{i=7}^{12} \left[\int_{T-}^{T} \left|\Gamma_i^-(t)\right| \mathrm{d}t + \int_{T}^{T+} \left|\Gamma_i^+(t)\right| \mathrm{d}t \right]. \tag{12}$$

With $\Gamma^-(t) = I^-\delta(t - T^-)$ and $\Gamma^+(t) = I^+\delta(t - T^+)$ the effort cost functional becomes:

$$\mathcal{W}_2^b = \sum_{i=1}^{6} \left[\left|I_i^-\right| + \left|I_i^+\right|\right], \quad \mathcal{W}_2^{ad} = \sum_{i=7}^{12} \left[\left|I_i^-\right| + \left|I_i^+\right|\right] \tag{13}$$

Three Criteria

Third Criterion:

- for the biped

$$\mathcal{W}_3^b = \max\left(\left|\Gamma_1^-(t)\right|, \left|\Gamma_2^+(t)\right|, \cdots, \left|\Gamma_i^-(t)\right|, \left|\Gamma_i^+(t)\right|, \cdots, \left|\Gamma_6^-(t)\right|, \left|\Gamma_6^+(t)\right|.\right. \quad (14)$$

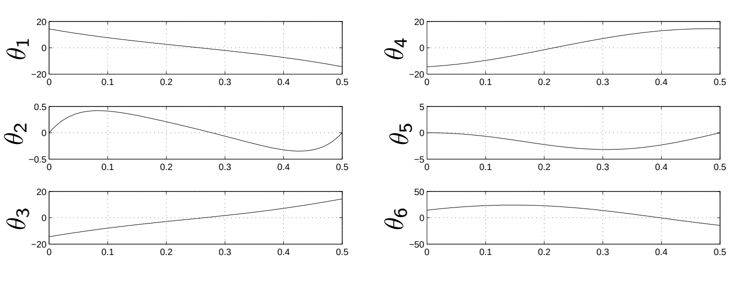
- for the assist device

$$\mathcal{W}_3^{ad} = \max\left(\left|\Gamma_7^-(t)\right|, \left|\Gamma_2^+(t)\right|, \cdots, \left|\Gamma_i^-(t)\right|, \left|\Gamma_i^+(t)\right|, \cdots, \left|\Gamma_{12}^-(t)\right|, \left|\Gamma_6^+(t)\right|.\right. \quad (15)$$

With $\Gamma^-(t) = I^-\delta(t - T^-)$ and $\Gamma^+(t) = I^+\delta(t - T^+)$ the effort cost functionals (14) and (15) become:

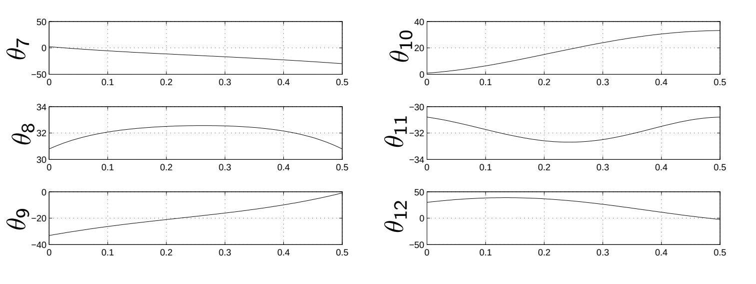
$$\begin{aligned} \mathcal{W}b_3 &= \max\left(\left|I_1^-(t)\right|, \cdots, \left|I_i^-(t)\right|, \cdots, \left|I_6^-(t)\right|\right), \\ \mathcal{W}_3^{ad} &= \max\left(\left|I_7^-(t)\right|, \cdots, \left|I_i^-(t)\right|, \cdots, \left|I_{12}^-(t)\right|\right). \end{aligned} \quad (16)$$

Joint variables of the locomotor system of the biped



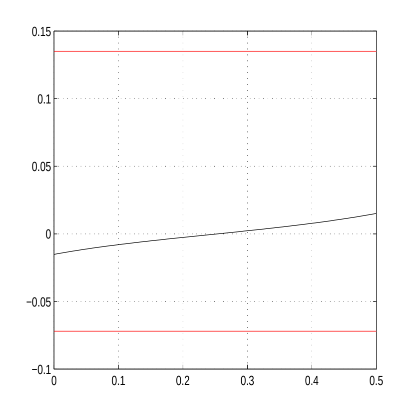
(a) (b)

Joint variables of the locomotor system of the assist device

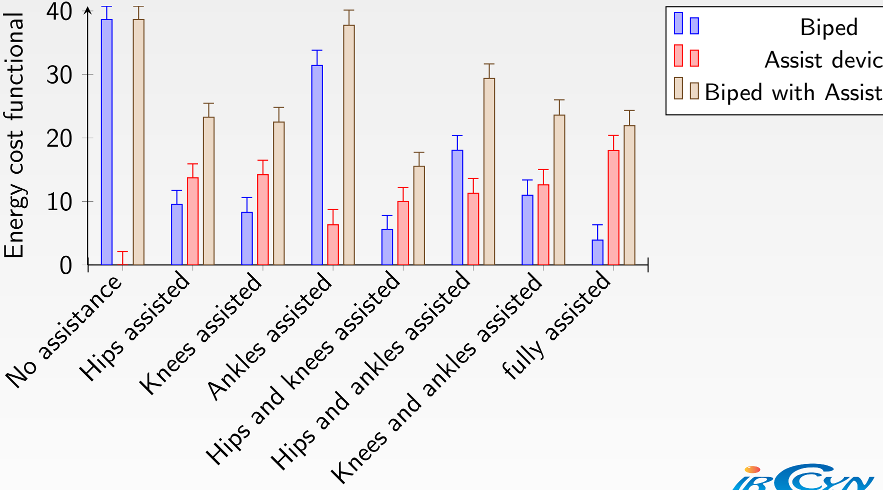


(c) (d)

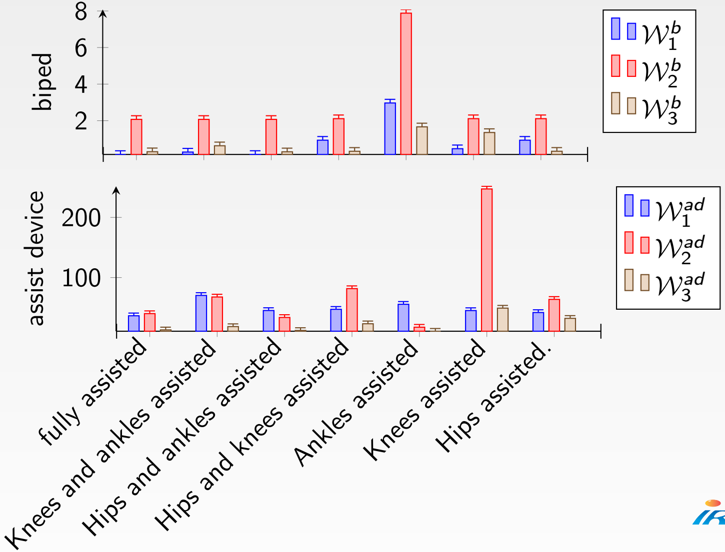
Position of the centre of pressure of the stance foot



Three distributions for the impulsive torques



Criteria as a function of Distributions of the impulsive torques



Conclusion

- Existence of a ballistic movement swinging for a biped with a wearable assist device for a given duration and length step.
- Impact is modeled with an instantaneous double support phase.
- Several solutions to design this phase $\Longrightarrow$: Statement of an optimization problem.
- During the instantaneous double support phase impulsive torques are applied.
- It is possible to compensate the efforts of the biped with the assist device.
- Perspectives:
  - Interaction between human and assistive device (spring, damping...).
  - To develop a new experimental assistive device (passive assist device?).